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Conformal Invariance in Weyl Gravity

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A conformal-invariant model of Weyl gravity, based on a nondecomposable representation of the conformal group, allows one to have a conformal-invariant propagator in an arbitrary gauge, as well as a conformal-invariant gauge-fixing term in the Lagrangian approach. It is shown that in the gauge-invariant sector this theory coincides with ordinary Weyl gravity (with conformal-noninvariant gauge fixing). The corresponding BRST transformations are found and are used for derivation of the Slavnov-Taylor identities.

1. INTRODUCTION

It has been proposed to use the nonbasic representations of the conformal group (CG) (Mack and Salam, 1969) in the case of conformal QED (Binegar *et al.*, 1983; Zaikov, 1983a, 1985; Furlan et al., 1983, 1985). In such a way the difficulties with the pure longitudinality of the conformal-invariant (CI) propagator and the impossibility of constructing a CI gauge-fixing term in the Lagrange approach are avoided. The corresponding generalization for the non-Abelian case was given in Zaikov (1983b, 1986a, b) and for conformal linear gravity in Furlan et al. (1986) and Zaikov (1987). Any model of CI QED considered in Binegar et al. (1983), Zaikov (1983a, 1985), and Furlan et al. (1983, 1985) (see also Petkova et al. (1985) and Sotkov et al. (1985) obeys the property that in the gauge-invariant sector (GIS) it coincides with ordinary QED (with conformal-noninvariant gauge). However, for the non-Abelian theory this is not the case, as is seen from the model considered in Zaikov (1983b). In Zaikov (1986a) a CI model of QED and the corresponding CI non-Abelian generalization are proposed, both of which have the above-mentioned property,

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i.e., in the GIS the non-Abelian model also coincides with the ordinary Yang-Mills theory.

In the present paper we consider a CI model of Weyl gravity [see Stelle (1977) for references to earlier papers on the subject]. In order to separate an appropriate CI model of linear gravity we use the same criterion as in the 4-vector case (Petkova et al., 1985; Zaikov, 1986a, b), i.e., when the self-interaction is included, our theory has to coincide in the GIS with ordinary (with nonconformal gauge), renormalizable (Stelle, 1977; Adler, 1982), asymptotically-free (Fradkin and Tzetlin, 1982) theory. Such a model of linear gravity was proposed in Zaikov (1987).

To clarify the problem, in Section 2 the results of Zaikov (1986a) (see also Zaikov, 1986b) are sketched. Section 3 considers the corresponding model of conformal linear gravity when only a 4-vector nonphysical field is included. This allows us to find nontrivial CI two-point functions as well as CI gauge-fixing. The corresponding self-interaction case is considered in Section 4. The Green's function generating functional is written in a local form using the Faddeev-Popov 4-vector "ghost" field with subcanonical scale dimension-1 (in mass units) transforming on the basic representation of the CG. It is shown (on a formal level) that in the GIS this theory coincides with ordinary Weyl gravity (Stelle, 1977). In Section 5 the BRST transformations are found and used to derive the Slavnov-Taylor identities.

2. CONFORMAL QED AND YANG-MILLS THEORY

To clarify the problem, let us fist review some of main results of the conformal QED and Yang-Mills theory obtained in Zaikov (1986a, b). These theories are based on the following effective CI Lagrangians:

$$\mathscr{L}_{\text{eff}} = \mathscr{L}_{\text{GI}} + \mathscr{L}_{\text{GE}} + \mathscr{L}_{\text{FP}} \tag{1}$$

where

$$\mathscr{L}_{\text{GI}} = \begin{array}{ll} -\dfrac{1}{4} F^{\mu\nu} F_{\mu\nu} & \text{for QED} \\ -\dfrac{1}{8g^2} \operatorname{tr}(G^{\mu\nu} G_{\mu\nu}) & \text{for YM} \end{array} \tag{1a}$$

$$\mathscr{L}_{\text{GE}} = \begin{cases} \dfrac{1}{2} \partial A \,\square R + (\partial^\mu A^\nu - \partial^\nu A^\mu)\, \partial_\mu R\, \partial_\nu S + \dfrac{1}{8} \alpha (\square R)^2 + \dfrac{1}{2} \nu (\square S)^2 \\ \dfrac{1}{2g^2} \operatorname{tr} \left[\dfrac{1}{2} \partial B \,\square \tilde{R} + (\partial^\mu B^\nu - {}^\nu B^\mu)\, \partial_\mu \tilde{R}\, \partial_\nu S + \dfrac{\alpha}{8} (\square \tilde{R})^2 \right] + \dfrac{\nu}{2} (\square S)^2 \end{cases} \tag{1b}$$

$$\mathscr{L}_{EP}=\begin{cases} 0 \\ \frac{1}{g^2}\operatorname{tr}(\bar{\zeta}\{\Box(\Box+\partial^\mu[B_\mu,\eta])-2\partial^\mu S(\eta_{\mu\nu}\Box-\partial_\mu\partial_\nu)[B^\nu,\zeta]\}) \end{cases} \tag{1c}$$

Here

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu, \qquad G_{\mu\nu}=\partial_\mu B_\nu-\partial_\nu B_\mu+[B_\mu, B_\nu]$$

$B_\mu = B^a_\mu T_a$, $\tilde{R} = R^a T_a$, T_a are the generators of the group $SU(N)$ in adjoint representation, and ζ and $\bar{\zeta}$ are scalar dimensionless Faddeev-Popov "ghost" fields. The Lagrangian (1) is CI when the fields A_μ (B_μ) and R are transformed according to the basic representations ($[\Phi(x), K_\lambda]_{x=0}=0$; $\Phi=A_\mu, B_\mu, R$) and $S(x)$ is the associated homogeneous field with the following conformal transformation laws:

$$\begin{aligned}
&[S(x), D]=ix^\mu\partial_\mu S(x)-\mathbb{1}\\
&[S(x), K_\lambda]=i(2x_\lambda x^\tau\partial_\tau-x^2\partial_\lambda)S(x)-2ix_\lambda\mathbb{1}
\end{aligned} \tag{2}$$

Here $\mathbb{1}$ is the identity operator, D the dilatational generator, and K_λ the special conformal generators.

Consequently, supposing that A_μ (B_μ) is a basic field and introducing the associated scalar field $S(x)$, which in the non-Abelian case is $SU(N)$ scalar also, we are able to construct the CI gauge-fixing term (1b). Note that $\mathscr{L}_{GF}$ in (1b) has the same form in both the Abelian and non-Abelian case. Moreover, in the gauge-invariant sector (GIS) these theories coincide with the ordinary (with nonconformal gauge-fixing) QED or the Yang-Mills theory, respectively (Zaikov, 1986a, b).

We recall that in the Abelian case we reached the conformal gauge-fixing using the nonbasic representations for the electromagnetic potential $\{[A_\mu(x), K_\lambda]_{x=0}=2i\eta_{\mu\lambda}R(0)\}$,[2] (Zaikov 1983a, 1985; Furlan *et al.*, 1983, 1985) for which the theory in the GIS coincides with the ordinary QED (Petkova et al., 1985). However, in the non-Abelian case (Zaikov, 1983b) it is not clear whether the theory with nonbasic field coincides with the ordinary Yang-Mills theory in the GIS.

In the latter models, when the gauge potentials are transformed according to the nonbasic representation of the CG, the CI two-point functions have nonzero transverse parts, which is not the case for the basic representations. To avoid this defect of the basic representations, it is assumed that the conformal symmetry is spontaneously broken in the following manner:

$$\delta_D|0\rangle=\delta\varepsilon\, U|0\rangle \tag{3}$$

$$\delta_c|0\rangle=\delta c^\mu\, V_\mu|0\rangle \tag{4}$$

[2] We call representations for which $[\Phi(x), K_\lambda]_{x=0}=0$ basic representations of the CG. The nonbasic representations, for which $[\Phi(x), K_\lambda]_{x=0}\neq 0$, are nondecomposable for any value of the scale dimension.

Here $\delta\varepsilon$ and δc_μ are the infinitesimal parameters of the dilatational and special conformal transformations, and U and V_μ are operators satisfying

$$\begin{aligned}
&[U, A_\mu(x)] = [U, R(x)] = 0 \\
&[U, S(x)] = \hat{q} - \mathbb{1} \\
&[V_\lambda, A_\mu] = 2i\eta_{\mu\lambda} R(x) \\
&[V_\lambda, R(x)] = 0 \\
&[V_\lambda, S(x)] = 2ix_\lambda(\hat{q} - \mathbb{1})
\end{aligned} \tag{5}$$

where $\hat{q}$ is a constant operator with the following properties:

$$\langle \hat{q}\hat{q}\rangle_0 = \langle \hat{q} A_\mu(x)\rangle_0 = \langle \hat{q} R(x)\rangle_0 = 0$$

$$\langle \hat{q} S(x)\rangle_0 = \text{const}$$

The operator $\hat{q}$ is introduced in (5) in order to get the CI two-point function for the field $S(x)$ in the form $\langle S(x)S(y)\rangle_0 \sim \ln[\mu^2(x-y)^2]$ (Sotkov and Stoyanov, 1980). If it is assumed that $[U, S(x)] = [V_\mu, S(x)] = 0$, then we have $\langle S(x)\rangle_0 \sim \ln \mu^2 x^2$, which is not translational invariant.

It can be checked that any CI N-point function for the gauge potential transformed on the basic representation $\{[A_\mu(x), K_\lambda]_{x=0} = 0\}$, with the vacuum state transformed according to (2) and (3), coincides with those when the gauge potential is transformed according to the nonbasic representation $\{[A_\mu(x), \tilde{K}_\lambda]_{x=0} = 2i\eta_{\mu\lambda}R(0)\}$. Indeed, for the two-point function we have

$$\begin{aligned}
&\delta_c(\langle 0|A_\mu(x)A_\nu(y)|0\rangle) \\
&\quad = -i\,\delta c^\lambda\{\langle 0|([A_\mu(x)A_\nu(y), V_\lambda] + [A_\mu(x)A_\nu(y), K_\lambda])|0\rangle\} \\
&\quad = -i\,\delta G^\lambda\{\langle 0|([A_\mu(x), K_\lambda] + 2i\eta_{\mu\lambda}R(x))A_\nu(y)|0\rangle \\
&\qquad + \langle 0|A_\mu(x)([A_\nu(y), K_\lambda] + 2i\eta_{\nu\lambda}R(y))|0\rangle\} \\
&\quad = -i\,\delta c^\lambda\langle 0|[A_\mu(x)A_\nu(y), \tilde{K}_\lambda]|0\rangle
\end{aligned}$$

where K_λ ($\tilde{K}_\lambda$) are the generators of the special conformal transformations in the basic (nonbasic) representation. Consequently, the rhs of (5) are chosen so that $[A_\mu(x), \tilde{K}_\lambda]_{x=0} = 2i\eta_{\mu\lambda}R(0)$, where $\tilde{K}_\mu = K_\mu + V_\mu$.

3. CONFORMAL LINEAR GRAVITY

Now, let us consider the following nondegenerate CI Lagrangian:

$$\mathscr{L}_{\mathrm{L}}(x) = \mathscr{L}_{\mathrm{lgr}} + \mathscr{L}_{\mathrm{GF}} \tag{6}$$

where

$$\mathscr{L}_{\mathrm{lgr}} = \tfrac{1}{2} h^{\mu\nu}(x) L^{\lambda\tau}_{\mu\nu}(\partial) h_{\lambda\tau}(x) = \tilde{\mathbb{C}}^{\mu\nu\lambda\tau} \tilde{\mathbb{C}}_{\mu\nu\lambda\tau} \tag{6a}$$

$$\mathscr{L}_{\mathrm{GE}} = \tfrac{1}{2} h^{\mu}_{(x)}(\delta^{\nu}_{\mu}\Box - \partial_{\mu}\partial^{\nu}) h_{\nu}(x) + h^{\mu}_{(x)} M^{\lambda\tau}_{\mu}(\partial) h_{\lambda\tau}(x) + \tfrac{1}{2} c (\Box S)^2 \tag{6b}$$

Here

$$L^{\lambda\tau}_{\mu\nu}(\partial) = [\tfrac{1}{2}\delta^{\{\lambda}_{\mu}\delta^{\tau\}}_{\nu} - \tfrac{1}{3}\eta_{\mu\nu}\eta^{\lambda\tau}]\Box^2 + \tfrac{1}{3}(\eta_{\mu\nu}\partial^{\lambda}\partial^{\tau} + \eta^{\lambda\tau}\partial_{\mu}\partial_{\nu})\Box$$
$$- \tfrac{1}{2}\delta^{\{\lambda}_{\{\nu}\partial_{\mu\}}\partial^{\lambda\}} + \tfrac{2}{3}\partial_{\mu}\partial_{\nu}\partial^{\lambda}\partial^{\tau} \tag{7}$$

$$M^{\lambda\tau}_{\mu}(\partial) = \tfrac{1}{4}(\delta^{\{\lambda}_{\mu}\partial^{\tau\}} + \tfrac{1}{6}\eta^{\lambda\tau}\partial_{\mu})\Box^2 - \tfrac{2}{3}\partial_{\mu}\partial^{\lambda}\partial^{\tau} - 2\partial^{\nu} S L^{\lambda\tau}_{\mu\nu} \tag{8}$$

$$\tilde{\mathbb{C}}^{\lambda\tau}_{\mu\nu} = \tfrac{1}{2}\partial^{[\lambda}\partial_{[\mu} h^{\tau]}_{\nu]} - \tfrac{1}{4}\delta^{[\lambda}_{[\mu}\Box h^{\tau]}_{\nu]} - 2\delta^{[\lambda}_{[\mu}\partial^{\nu]}\partial_{\rho} h^{\rho}_{\tau]}$$
$$- \tfrac{1}{6}\delta^{\lambda}_{[\mu}\delta^{\tau}_{\nu]}\Box h^{\rho}_{\rho}(x) \tag{9}$$

The symbol {,} ([,]) denotes symmetrization (antisymmetrization), $\mathbb{C}^{\lambda\tau}_{\mu\nu}$ is the linearized Weyl tensor, $\eta_{\mu\nu}(\operatorname{diag}\eta_{\mu\nu} = (1, -1, -1, -1))$ is the metric tensor in the Minkowski space-time, $h_{\mu\nu}(x)$ is a symmetric, traceless, and dimensionless tensor field transforming according to the basic representation of the CG

$$[h_{\mu\nu}(x), K_{\lambda}] = i\{(2x_{\lambda}x^{\tau}\partial_{\tau} - x^2\partial_{\lambda}) h_{\mu\nu}(x)$$
$$+ 2ix^{\tau}[(\Sigma_{\lambda\tau})^{\rho}_{\mu} h_{\rho\nu}(x) + (\Sigma_{\lambda\tau})^{\rho}_{\nu} h_{\mu\rho}(x)]\} \tag{10}$$

$h_{\mu}(x)$ is a basic 4-vector field with subcanonical scale dimension -1 (in mass units), which is transformed according to the law

$$[h_{\mu}(x), K_{\lambda}] = i\{[2x_{\lambda}(-1 + x^{\tau}\partial_{\tau}) - x^2\partial_{\lambda}] h_{\mu} + 2ix^{\tau}(\Sigma_{\lambda\tau})^{\nu}_{\mu} h_{\nu}\} \tag{11}$$

and $S(x)$ is the associated homogeneous scalar field transforming according to the conformal laws (2), β is an arbitrary gauge parameter, and c is another parameter.

Then it can be checked that (6) is conformal-invariant. Moreover, $\mathscr{L}_{\mathrm{lgr}}$ in (6), as well as any term of (6b), is separately CI.

The interaction with a matter field can be included trough the energy-momentum tensor of the matter field $T_{\mu\nu} = T_{\nu\mu}$, $T^{\mu}_{\mu} = 0$:

$$\mathscr{L}_{\mathrm{int}} = h^{\mu\nu}(x) T_{\mu\nu}(x) \tag{12}$$

It can be checked that (12) is CI if $T_{\mu\nu}$ is again a basic field with scale dimension 4.

Let us sketch the proof of the conformal-invariance of the Lagrangian (6). For this purpose we introduce the fields ${}^{\pm}h_{\mu\nu}(x)$ with the following nonbasic transformation laws:

$$[{}^{\pm}h_{\mu\nu}(x), K_{\lambda}]_{x=0} = \pm 2i\{\eta_{\mu\lambda} h_{\nu}(0) + \eta_{\nu\lambda} h_{\mu}(0) - \tfrac{1}{2}\eta_{\mu\nu} h_{\lambda}(0)\} \tag{13}$$

Then the Lagrangian (6) can be represented in the following way:

$$\mathscr{L}_{\mathrm{L}} = (h_\mu, {}^+h_{\mu\nu})^{++}G^{-1}(\partial)\begin{pmatrix} h^\lambda \\ {}^+h^{\lambda\tau}\end{pmatrix} - (h_\mu, {}^-h_{\mu\nu})^{--}G^{-1}(\partial)\begin{pmatrix} h^\lambda \\ {}^-h^{\lambda\tau}\end{pmatrix} \quad (6c)$$

where

$$^{(\pm\pm)}G^{-1}(\partial)$$

are intertwining operators (CI two-point function) for the representations ${}^\pm\lambda = \{\pm, \{-1, 0\}, \{1, 2\}\}$ and ${}^\pm\varkappa = \{\pm, \{5, 4\}, \{1, 2\}\}$,

$${}^\pm G^{-1} T_{{}^\pm\lambda} = T_{{}^\pm\varkappa}{}^\pm G^{-1}$$

Here ${}^\pm\chi$ denotes the nonbasic representation for the fields $\{h_\mu, {}^\pm h_{\mu\nu}\}$ and ${}^\pm\varkappa$ denotes the nonbasic representation for the fields $\{{}^\pm t_\mu, T_{\mu\nu}\}$ with transformation laws

$$[T_{\mu\nu}(x), K_\lambda]_{x=0} = 0, \qquad [{}^\pm t_\mu(x), K_\lambda]_{x=0} = \mp 2iT_{\mu\lambda}(0) \quad (14)$$

Then, inserting in (6c)

$$h_{\mu\nu}(x) = {}^+h_{\mu\nu}(x) + {}^-h_{\mu\nu}(x) \quad (15)$$

we find the CI Lagrangian (6). As follows from (13), the field (15) obeys exactly the law (10).

Now, in order to have nontrivial CI two-point functions (with nonzero transverse part), we suppose, as in the 4-vector case, noninvariance of the vacuum state. For the vacuum state the transformation laws (3) and (4) are adopted, where the commutation rules (5) are completed with

$$\begin{aligned} &[h_{\mu\nu}(x), U] = [h_\mu(x), U] = 0 \\ &[h_{\mu\nu}(x), V_\lambda] = -2i[\eta_{\mu\lambda}h_\nu(x) + \eta_{\mu\lambda}h_\mu(x) - \tfrac{1}{2}\eta_{\mu\nu}h_\lambda(x)] \\ &[h_\mu(x), V_\lambda] = 0 \end{aligned} \quad (16)$$

In the same way as in the 4-vector case, it can be checked that the CI two-point functions found here coincide with those for the fields ${}^+h_{\mu\nu}$ with CI vacuum state. The Fourier transform of the corresponding time-order CI two-point function is given by

$$\begin{aligned} D_c(p) &= \int d^4x\, l^{ipx}\langle 0|T\left\{(h^\mu_{(x)}, h^{\mu\nu}_{(x)})\begin{pmatrix} h_\lambda(0) \\ h_{\lambda\tau}(0)\end{pmatrix}\right\}|0\rangle \\ &= \begin{pmatrix} (D_c(p))^\mu_\lambda (D_c(p))^\mu_{\lambda\tau} \\ (D_c(p))^{\mu\nu}_\lambda (D_c(p))^{\mu\nu}_{\lambda\tau}\end{pmatrix} \end{aligned} \quad (17)$$

where

$$(D_c(p))^{\mu}_{\lambda} = \frac{\pi^2}{16}\left(\delta^{\mu}_{\lambda}\frac{\partial^2}{\partial p^{\tau}\partial p_{\tau}} + 2\frac{\partial^2}{\partial p_{\mu}\partial p^{\lambda}}\right)\delta^4(p)$$

$$(D_c(p))^{\mu}_{\lambda\tau} = \frac{2i}{(p^2+i\varepsilon)^4}(6p^{\mu}p_{\lambda}p_{\tau} - \eta_{\lambda\tau}p^{\mu}p^2 - \delta^{\mu}_{\lambda}p_{\tau}p^2 - \delta^{\mu}_{\tau}p_{\lambda}p^2)$$

$$\begin{aligned}(D_c(p))^{\mu\nu}_{\lambda\tau} = {} & \frac{1}{(p^2+i\varepsilon)^4}\left\{\left(\frac{1}{2}\delta^{\{\mu}_{\lambda}\delta^{\nu\}}_{\tau} - \frac{3+\beta}{4}\eta^{\mu\nu}\eta_{\lambda\tau}\right)(p^2+i\varepsilon)^2\right.\\ & + \left[(2+\beta)(\eta^{\mu\nu}p_{\lambda}p_{\tau} + \eta_{\lambda\tau}p^{\mu}p^{\nu}) + \frac{\beta-1}{2}\delta^{\{\mu}_{\{\lambda}p^{\nu\}}p_{\lambda\}}\right]p^2\\ & \left. + (10-6\beta)p^{\mu}p^{\nu}p_{\lambda}p_{\tau}\right\}\\ & + \frac{\pi^2\beta}{24}(\delta^{\{|}_{\lambda}\delta^{\nu\}}_{\tau} - \frac{1}{2}\eta^{\mu\nu}\eta_{\lambda\tau})\delta^4(p)\end{aligned} \tag{17a}$$

Here β is an arbitrary gauge parameter.

It can be checked that $\mathscr{L}_{\rm lgr}$ in (7) is invariant with respect to the following Abelian gauge transformations:

$$h_{\mu\nu}(x) \to h_{\mu\nu}(x) + \partial_{\mu}\xi_{\nu}(x) + \partial_{\nu}\xi_{\mu}(x) - \tfrac{1}{2}\eta_{\mu\nu}\partial^{\lambda}\xi_{\lambda} \tag{18}$$

where $\xi_{\mu}(x)$ is an arbitrary 4-vector function with scale dimension -1.

We also note that $\mathscr{L}_{\rm GF}$ in (6) is one of the admissible gauge fixings. Indeed, integration over the nonphysical fields $h_{\mu}(x)$ and $S(x)$ gives

$$\begin{aligned}F(h_{\mu\nu}) &= \int Dh_{\mu}(x)DS(x)\exp i\int d^4x\,[\mathscr{L}_{\rm GF} + h^{\mu}J_{\mu} + SJ(x)]\\ &= \int DS(x)\exp i\int d^4x\left\{\frac{c}{2}(\Box S)^2 + SJ + \int d^4y\frac{1}{2\beta}[M^{\mu}_{\lambda\tau}h^{\lambda\tau}\right.\\ &\quad \left. + J^{\mu}(x)]^4D^{\nu}_{\mu}(x-y)[M^{\sigma\rho}_{\nu}h_{\sigma\rho} + J_{\nu}(y)]\right\}\\ &= \int DS(x)\exp i\int d^4x\left[\frac{c}{2}(\Box S)^2 + SJ(x)\right]F(h_{\mu\nu}, S, J)\end{aligned} \tag{19}$$

Here $J_{\mu}(x)$ and J are sources,

$${}^4D^{\nu}_{\mu}(x) = (\delta^{\nu}_{\mu}\Box - 3\partial_{\mu}\partial^{\nu})D_4(x)$$

where $D_4(x)$ is the Green's function of the equation $\Box^4 f(x) = 0$, i.e., $\Box^4 D_4(x) = -\delta^{(4)}(x)$.[3] It is evident that the integral over $S(x)$ can be taken

[3]The formal integrals in (19) can be defined using the conformal operator product expansion (Petkova *et al.*, 1985).

only in a perturbative way. The corresponding Faddeev–Popov determinant is given by

$$
\begin{aligned}
&\Delta^{-1}(J_\mu=0, H=0)\\
&=\int D\xi_\mu(x)\, F\left(h_{\mu\nu}+\partial_\mu\xi_\nu+\partial_\nu\xi_\mu-\frac{1}{2}\,\eta_{\mu\nu}\partial^\lambda\xi_\lambda\right)\\
&=\int DS(x)\, D\xi_\mu(x)\exp i\int d^4x\left\{\frac{c}{2}(\Box S)^2\left[\int d^4y\frac{1}{2\beta}\left(\frac{1}{2}\,\delta^\nu_\mu\Box\right.\right.\right.\\
&\quad\left.\left.\left.-\frac{3}{4}\partial_\mu\partial^\nu\right)\Box\,\xi^4_\nu D^\mu_\lambda(x-y)\left(\frac{1}{2}\,\delta^\lambda_\tau\Box-\frac{3}{4}\partial^\lambda\partial_\tau\right)\Box\,\xi^\tau(y)\right]\right\}=\text{const}
\end{aligned}
$$

where the following substitution was performed:

$$
\begin{aligned}
\xi_\mu(x)\to\xi_\mu(x)&-(2\delta^\nu_\mu\Box-6\partial_\mu\partial^\nu)\,\Box^{-3}[\tfrac{1}{4}(\delta^\lambda_\nu\partial^\tau+\tilde{\partial}^\tau_\nu\partial^\lambda+\tfrac{1}{6}\eta^{\lambda\tau}\partial_\nu)\\
&-\tfrac{2}{3}\partial_\nu\partial^\lambda\partial^\tau-2\partial^\rho SL^{\lambda\tau}_{\nu\rho}(\partial)]h_{\lambda\tau}(x)
\end{aligned}
$$

Consequently, as in the conformal QED and Yang–Mills theory, in GIS the theory is equivalent to the corresponding theory with nonconformal gauge (for instance, $\partial^\mu h_{\mu\nu}=0$).

4. SELF-INTERACTION THEORY

Consider the following CI effective Lagrangian:

$$
\mathscr{L}(x)=\mathscr{L}_{\text{gr}}+\mathscr{L}_{\text{GF}}+\mathscr{L}_{\text{FP}} \tag{20}
$$

Here $\mathscr{L}_{\text{gr}}$ can be found from (6a) by substituting the linearized Weyl tensor with the full Weyl tensor,

$$
\begin{aligned}
\tilde{\mathbb{C}}_{\mu\nu\lambda\tau}&\to\mathbb{C}_{\mu\nu\lambda\tau}\\
&=R_{\mu\nu\lambda\tau}-\tfrac{1}{2}[g_{\mu\lambda}(x)R_{\nu\tau}+g_{\nu\tau}(x)R_{\mu\lambda}-g_{\mu\tau}(x)R_{\nu\lambda}-g_{\nu\lambda}(x)R_{\mu\tau}]\\
&\quad+\tfrac{1}{6}[g_{\mu\lambda}(x)g_{\nu\tau}(x)-g_{\mu\tau}(x)g_{\nu\lambda}(x)]
\end{aligned} \tag{21}
$$

i.e.,

$$
\mathscr{L}_{\text{gr}}=\mathbb{C}^{\mu\nu\lambda\tau}\mathbb{C}_{\mu\nu\lambda\tau} \tag{22}
$$

Here $R_{\mu\nu\lambda\tau}$ is the Ricci tensor, $R_{\mu\nu}$ the curvature tensor, R the scalar curvature, and

$$
g_{\mu\nu}(x)=h_{\mu\nu}(x)+\eta_{\mu\nu} \tag{23}
$$

is the metric tensor in Riemannian space-time. As is well known, the Lagrangian (22) is invariant with respect to the metric transformation $g_{\mu\nu}(x)\to\omega(x)g_{\mu\nu}(x)$. This fact allow us to impose the constraint $\det|g_{\mu\nu}|=1$, and consequently, $h_{\mu\nu}(x)$ in (23) becomes a traceless tensor as in the linearized case. The term $\mathscr{L}_{\mathrm{GF}}$ in (20) has the same form (6b) as in the linearized case and the Faddeev–Popov term $\mathscr{L}_{\mathrm{FP}}$ is given by

$$\mathscr{L}_{\mathrm{FP}}(x) = \bar{\Theta}^{\mu}(x)\mathscr{M}_{\mu}^{\nu}\Theta_{\nu}(x) \tag{24}$$

where $\Theta_{\mu}(x)$ is anticommuting Faddeev–Popov 4-vector "ghost" field whose conformal transformation law is the same as for the field $h_{\mu}(x)$,

$$\begin{aligned}\mathscr{M}_{\mu}^{\nu} &= [\tfrac{1}{4}(\delta_{\mu}^{\{\lambda}\delta^{\tau\}} + \tfrac{1}{6}\eta^{\lambda\tau}\partial_{\mu})\Box^{2} - \tfrac{2}{3}\partial_{\mu}\partial^{\lambda}\partial^{\tau}\Box - 2\partial^{\rho}SL_{\mu\rho}^{\lambda\tau}]D_{\lambda\tau}^{\nu} \\ &= M_{\mu}^{\lambda\tau}D_{\lambda\tau}^{\nu}\end{aligned} \tag{25}$$

and

$$D_{\mu\nu}^{\lambda} = \delta_{\{\mu}^{\lambda}\partial_{\nu\}} - \eta_{\mu\nu}\partial^{\lambda} + \delta_{\mu}^{\lambda}h_{\tau\nu}\partial^{\tau} + \delta_{\nu}^{\lambda}h_{\mu\tau}\partial^{\tau} - \partial^{\lambda}h_{\mu\nu} - h_{\mu\nu}\partial^{\lambda}$$

is the covariant derivative.

The Weyl tensor $\mathbb{C}_{\mu\nu\lambda\tau}$ is transformed with respect to the conformal transformations in the same way as $\tilde{\mathbb{C}}_{\mu\nu\lambda\tau}$ and consequently the CI of $\mathscr{L}_{\mathrm{gr}}$ and $\mathscr{L}_{\mathrm{GF}}$ is evident.

To prove the CI of $\mathscr{L}_{\mathrm{FP}}$, we use that $D_{\mu\nu}^{\rho}\Theta_{\rho}$ is transformed in the same way as $h_{\mu\nu}$ (if Θ_{ρ} and Θ_{ρ} are basic fields with scale dimension -1), i.e.,

$$[D_{\mu\nu}^{\rho}\Theta_{\rho}, K_{\lambda}]_{x=0} = 0$$

which can be checked by direct computation. Then the CI of the Faddeev–Popov term is implied by the equality

$$\mathscr{L}_{\mathrm{FP}} = \mathscr{L}_{\mathrm{GF}}(h_{\mu} \to \bar{\Theta}_{\mu},\, h_{\mu\nu} \to D_{\mu\nu}^{\rho}\Theta_{\rho}) - \tfrac{1}{2}h^{\mu}(\delta_{\mu}^{\nu}\Box - \tfrac{3}{2}\partial_{\mu}\partial^{\nu})\Box^{2}h_{\nu}$$

which follows from (23) and (25) if it is taken into account that the term $\frac{1}{2}h^{\mu}(\delta_{\mu}^{\nu}\Box - \partial_{\mu}\partial^{\nu})\,\Box^{2}h_{\nu}(x)$ itself is CI.

The Lagrangian $\mathscr{L}_{\mathrm{gr}}$ (22) is invariant with respect to the non-Abelian gauge transformations:

$$\delta h^{\mu\nu}(x) = D_{\lambda}^{\mu\nu}h^{\lambda}(x) \tag{26}$$

induced by the coordinate transformations $\delta x_{\mu} = h_{\mu}(x)$.

Now, let us write down the Green's function generating functional:

$$\begin{aligned}&Z(J_{\mu\nu}, J_{\mu}, J, h, \bar{h}) \\ &\quad = \int Dh_{\mu\nu}\,Dh_{\mu}DS\,D\Theta\,D\bar{\Theta} \\ &\qquad \times \exp i\int d^{4}x\,[\mathscr{L}(x) + h^{\mu\nu}(x)J_{\mu\nu}(x) + h^{\mu}(x)J_{\mu}(x) + S(x)J(x) \\ &\qquad + \bar{\Theta}h + \bar{h}\Theta(x)] \\ &\overset{h=\bar{h}=0}{=} \int Dh_{\mu\nu}\,DS\,\exp i\int d^{4}x\,[\mathscr{L}_{\mathrm{gr}}(x) + h^{\mu\nu}(x)J_{\mu\nu}(x) + SJ(x)] \\ &\qquad \times F(h_{\mu\nu}, S, J_{\mu})\,\Delta(h_{\mu\nu}, S)\end{aligned} \tag{27}$$

where $J_{\mu\nu}(x)$, $J_\mu(x)$, $J(x)$, $h(x)$, and $\bar{h}(x)$ are sources, F is given by (19), and Δ is the Faddeev-Popov determinant

$$\Delta = \det\{\mathcal{M}(h_{\mu\nu}, S)\} \tag{28}$$

In order to prove the identity

$$N \int DS(x) \int Dh_\mu(x) F(h_{\mu\nu} + \delta h_{\mu\nu}, S, \Theta)\tilde{\Delta}(h_{\mu\nu} + \delta h_{\mu\nu}, S) = 1 \tag{29}$$

where $\tilde{\Delta}$ is the Faddeev-Popov determinant corresponding to the $F(h_{\mu\nu}, S)$, we use the representation of F given by (19)

$$F(h_{\mu\nu}, S, J_\mu) = \exp i(\mathcal{F})^2 = \int Da(x)\, e^{ia^2(x)}\delta(\mathcal{F} - a) \tag{30}$$

It is easy to check that

$$\begin{aligned}\mathcal{F}'(h_{\mu\nu}, S, J_\mu) &= \not{\partial}[\delta^\nu_\mu \Box - (1 + i\sqrt{2})\partial_\mu \partial^\nu]\mathcal{F}(h_{\mu\nu}, S, J_\mu)\\ &= \not{\partial}[\delta^\nu_\mu \Box - (1 + i\sqrt{2})\partial_\mu \partial^\nu](M^{\lambda\tau}_\nu h_{\lambda\tau} + J_\nu)\end{aligned} \tag{31}$$

Here the operator $M^{\lambda\tau}_\mu$ is given by (8) and $\not{\partial} = \gamma^\mu \partial_\mu$ is the Dirac operator arising as a square root of the D'Alambert operator $\Box$.

Now, using the Faddeev-Popov trick, it can be checked that in the GIS the substitution

$$F\tilde{\Delta} \to \mathcal{F}\Delta \tag{32}$$

does not change the generating functional (27). Let us remark that the determinant corresponding to $\mathcal{F}'$ in (31) differs from Δ in (28) by a multiplicative constant. Then, it is evident that in the GIS, (27) is equivalent to the corresponding generating functional with nonconformal gauge, for instance, the harmonic gauge $\partial^\mu h_{\mu\nu} = 0$. This allows us to use the results of Stelle (1977) on the renormalizability and unitarity of the theory under consideration.

5. BRST SYMMETRY AND SLAVNOV–TAYLOR IDENTITIES

As a whole, the Lagrangian (20) is not invariant with respect to the gauge transformations (26). However, as in the ordinary case (Stelle, 1977), there is a remaining BRST symmetry with respect to the transformations:

$$\begin{aligned}\delta h_{\mu\nu}(x) &= \delta\lambda\,(s h_{\mu\nu}) = \delta\lambda\, D^\rho_{\mu\nu}\Theta_\rho(x)\\ \delta h_\mu(x) &= \delta\lambda\,(s h_\mu) = 0\\ \delta S(x) &= \delta\lambda\,(sS) = 0\\ \delta\Theta_\mu(x) &= \delta\lambda\,(s\Theta_\mu) = -\delta\lambda\, \partial^\nu\Theta_\mu(x)\Theta_\nu(x)\\ \delta\bar{\Theta}_\mu(x) &= \delta\lambda\,(s\bar{\Theta}) = -\delta\lambda h_\mu(x)\end{aligned} \tag{33}$$

Indeed, the term $\mathscr{L}_{gr}$ in (22) is invariant with respect to (33) because $\delta h_{\mu\nu}$ is a special kind of gauge transformation (26). It can be checked that

$$\delta(D^{\lambda}_{\mu\nu}\Theta_{\lambda}) = \delta(\delta h_{\mu\nu}) = \delta(\delta\Theta) = \delta(\delta\bar{\Theta}) = 0 \tag{34}$$

which follows from the nilpotency of the transformations (33). Then, from the equality

$$\delta\mathscr{L}_{FP} = \delta\bar{\Theta}^{\mu}\mathscr{M}^{\nu}_{\mu}\Theta_{\nu} = \delta\bar{\Theta}^{\mu}M^{\lambda\tau}_{\mu}D^{\nu}_{\lambda\tau}\Theta_{\nu} = -\delta\mathscr{L}_{GF}$$

we obtain the invariance of the Lagrangian (20) with respect to the BRST transformations (33).

In order to derive the Slanov-Taylor identities, let us write the one-particle irreducible Green's function generating functional:

$$\begin{aligned}
&\Gamma(h_{\mu\nu}, h_{\mu}, \Theta_{\mu}, \bar{\Theta}_{\mu}, K_{\mu\nu}, \bar{L}_{\mu}) \\
&\quad = G(\tau_{\mu\nu}, \tau_{\mu}, \chi_{\mu}, \bar{\chi}_{\mu}, K_{\mu\nu}, L_{\mu}, \bar{L}_{\mu}) \\
&\quad\quad - \int d^4x\,[\tau^{\mu\nu}h_{\mu\nu} + \tau^{\mu}h_{\mu} + \bar{\Theta}^{\mu}\chi_{\mu} + \bar{\chi}^{\mu}\Theta_{\mu} + K^{\mu\nu}(sh_{\mu\nu}) + L^{\mu}(s\Theta)_{\mu} \\
&\quad\quad + \bar{L}^{\mu}(s\bar{\Theta}_{\mu})]
\end{aligned} \tag{35}$$

where $\tau_{\mu\nu}$, τ_{μ}, χ_{μ}, and $\bar{\chi}_{\mu}$ are sources of the fields $h_{\mu\nu}$, h_{μ}, Θ_{μ}, and Θ_{μ} and $K_{\mu\nu}$, L_{μ}, and $\bar{L}_{\mu}$ are sources associated with $sh_{\mu\nu}$, $s\Theta_{\mu}$, and $s\Theta_{\mu}$.

Taking into account that the Lagrangian (20) is invariant with respect to the BRST transformations (33), from (35) we find the following Slanov-Taylor identities:

$$\frac{\delta\Gamma}{\delta h^{\mu\nu}}\frac{\delta\Gamma}{\delta K_{\mu\nu}} + \frac{\delta\Gamma}{\delta\Theta^{\mu}}\frac{\delta\Gamma}{\delta L_{\mu}} + \frac{\delta\Gamma}{\delta\bar{\Theta}^{\mu}}\frac{\delta\Gamma}{\delta\bar{L}_{\mu}} = 0 \tag{36}$$

For deriving the ST identities (36) the nilpotency property (34) of the BRST transformations is used. The conformal invariance of the ST identities (36) follows from the commutativity of the conformal transformations (10) and (11) with the BRST transformations (33). Moreover, the identities (36) are found in symmetric form without using the "ghost" equations of motion (Stelle, 1977).

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